Amending Kepler's Third Law

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Abstract

Kepler's third law of planetary motion needs to be amended.
First step:
Kepler's Third Law of Planetary Motion has an equation which apparently mixes units. This paper explains how this equation actually works.
Second step: Explain how this change affects the broader scope of moons and exoplanets.

Contents

1 Introduction

Celestial Mechanics explains the motions of the planets in our solar system. This should include the motions of moons around planets and of exoplanets around other stars.

Kepler's 3 laws of planetary motion are an essential part of celestial mechanics. These laws describe elliptical orbits. These orbits exist beyond the scope of planets in our solar system.
Kepler's Third Law of Planetary Motion has an equation which mixes different units (astronomical unit and year).
This paper explains how this equation in the 3rd law works.

2 Celestial Mechanics

Wikipedia has a basic description:
Celestial mechanics is the branch of astronomy that deals with the motions of objects in outer space. Historically, celestial mechanics applies principles of physics (classical mechanics) to astronomical objects, such as stars and planets, to produce ephemeris data.
This paper is about planets (both around our Sun and around other stars), asteroids, and moons but not stars. Stars in spiral galaxies do not have an elliptical motion around the galaxy center with increasing distances and times for their individual orbits where all are roughly concentric. The galactic magnetic field is the primary driver for the spiral rotation and the magnetic fields in the spiral arms provide the observed structure. Stars in elliptical galaxies or in globular clusters have radial orbits totally unlike a spiral galaxy and have a different mechanism.
Kepler's laws of planetary motion do not apply to stars. Stellar motion is not relevant to this paper.

3 Kepler's Third Law

Kepler's Third Law is the focus of this paper.

3.1 Its Description

excerpt from Wikipedia as a reference:

[Third Law:] The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

reference

web-link: Kepler's laws of planetary mption

3.2 Its Equation

This law can be expressed by the formula: $\mathrm{T}^2 = \mathrm{R}^3$ where T is the time or period in years and R is the radius in AU.

For examples:

a) Mercury T=0.2409 year and R= 0.3871 AU

b) Venus T=0.6152 year and R= 0.5352 AU

c) Earth T=1 year and R= 1 AU

These values conform to the equation.

The formula can be represented also as a a ratio, like:

$X = T^2 / R^3$

so X should equal 1.0
if X is >1.0 then T is too high for R, so one or both are wrong for a valid ellipse. In other words, the pair of values are not the correct proportions. Because published values are usually specified (or estimated) with a certain number of significant digits, the range of X is often from 0.998 to 1.002, or the two values (from squared and cubed) are very close to equality.

All the planets in the solar system conform to this equation.

3.3 Its Problem with units

The units in this equation cannot be $\text{year}^2 = \text{AU}^3$ because mixing these inconsistent units is invalid.

The current description lacks an explanation for the units.

3.4 Solving the problem with units

The values in the equation are actually ratios with respect to Earth, so as a ratio each value has no units. The description omits this detail. The equation simply demonstrates the orbits in the system are proportional to each other by using ratios.

3.5 New Equation

This new simple equation has unit-less values from ratios using the ratio of the planet's orbit to Earth's orbit.

This equation can be expressed in a slightly different format:

$TF^2 = RF^3$

where TF is the Time Factor and RF is the Radius Factor.

TF is the scalar factor relative to the baseline time, or orbital period, with the ratio of two values in whatever units are used to describe the period. The values from each planet could be years, days, hours, or anything consistent. This is a ratio where its common units are critical.

The ratios require unit consistency but, as a result, the equation uses values with no units.

RF is the scalar factor relative to the baseline radius from a ratio in whatever units are used to describe the radius. The values from each planet could be AU, km, or anything consistent. This factor is from a ratio.

Currently these factors are the ratio for each planet's orbit to Earth's orbit as the baseline.

That is why the numbers of years and of AU work.This equation can also use ratio values from a different planet as the basis than Earth.

The simple change: Instead of entering a planet's orbit values their proportional values are entered.

3.6 Confirming the solution

All of the planets could have their factor based on their ratio to another planet because all their ellipses are proportionally consistent. This equation is based on the ratios between the respective elliptical orbits.

Each factor is a ratio from a body's value divided by a baseline value. Currently we use Earth for the baseline for planets simply for its convenience.

For Mercury based on Earth, TF= 0.3871 yr / 1 yr or 0.3871 and RF = 0.3871 (from 0.3871 AU / 1 AU).
For Mercury based on Venus, TF = 0.3915 (from 0.2409 yr / 0.6152 yr). RF= 0.5352 (from 0.3871 AU / 0.7233 AU)

These proportional values based on Venus for Mercury conform to the $TF^2 = RF^3$ expectation.
As required, there are no units with the ratio values entered in the equation.
The interesting application of this new definition of Kepler's 3rd Law is its application to systems of moons.

3.7 Applying the solution to moons

Nearly every planet in our solar system has one or more moons, as well as dwarf planet Pluto; only Mercury and Venus have none.

As an initial test, Jupiter's set of moons can be verified with this new description.

Jupiter has a number of large moons: 1 Io, 2 Europa, 3 Ganymede, 4 Callisto, 5 Amalthea, 6 Himalia, 7 Elara

If the moons other than Europa have their TF and RF based on the orbit of Europa. this squared=cubed equation applies to all these moons. Europa can have its ratios based on Io.

The new description applies to Jupiter's moons.

This description change allows verification of orbits of moons as well as planets.

4 Applying this solution

The orbit parameters were entered into an Excel spreadsheet for all bodies in the solar system which could be found with this data (radius and time per orbit).

This included all planets, all their moons and most identified asteroids. Apparently only about 7000 of the (estimated) billions of asteroids have been identified but not all have their data published in a public archive like Wikipedia.

All the planets and asteroids in orbit around the Sun and all the moons were checked. This process was time consuming to enter the data and then verify their correct entry with the calculation using an appropriate baseline orbit for all the moons. The spreadsheet is a useful reference.

Earth has only 1 Moon but its orbit is roughly proportional to the International Space Station in its varying orbit (with distinct apogee and perigee).

Mars has 2 moons. Each can be the baseline for the other; both conform.

Jupiter has over 50 moons. Wikipedia notes some moons get lost and found again; 2 were found in 2004 and 2007 but both (unnamed) remain lost at the time of this paper.

Europa can serve as the baseline for most of the moons. Only a few were not closely proportional to Europa. Carme has one of the longest periods among Jupiter's moons. Harpalike provides a baseline. Both Megaclite and Chaldene have high eccentricities and Ananke provides a baseline.

The numbering of Saturn's moons is different in Brittanica or Wikipedia. This paper uses the Brittanica numbering.

Brittanica reference:

web-link: Brittanica list of Saturn's moons

Wikipedia reference:

web-link: Wikipedia list of Saturn's moons

In the case of Saturn's moons, the orbit of Janus can be the baseline for most moons. Janus can use Mimas for its baseline.

An issue with Saturn's moon collection is after the main moons there are more moons with higher eccentricites. As Wikipedia describes: Saturn has 24 regular satellites in prograde orbits not inclined. The remaining 58 are irregular satellites with a mix of prograde and retrograde. An unnamed moon found in 2004 is not closely proportional to Janus. This one is proportional to another unnamed one found in 2004.

In the case of moons around Uranus, the orbit of Miranda can be the baseline for most moons. Miranda can use Ophelia for its baseline. Uranus also has irregular satellites. Only 3 long period moons, Caliban, Sycorax, and Margaret are not closely proportional to Miranda. these 3 are proportional to similar lon period moons like Trincolo.

In the case of moons around Neptune, the orbit of Naiad can be the baseline for most moons. Naiad can use Thalassa for its baseline. Only 2 high eccentricity moons Nereid and Psamathe are not closely proportional to Naiad. Both can use Halimede which also has high eccentricity for their baseline.

In the case of 5 moons around Pluto, the orbit of Styx can be the baseline for Charon. Styx can use Charon for its baseline. Nix, Kerberos and Hydra are more closely proportional to any in this set of 3 than to Styx.

2 of the high eccentricity moons (Nereid and Psamathe) are not closely proportional to Styx. Nereid is closely proportional to Halimede, while Psamthe is closely proportional to Nereid. Apparently there are about 50 Nereids in a group around Pluto but only 3 (these 2 and Neso) have a name and data.
reference for this spreadsheet of orbit data:

pdf-link: Solar System Data with check of Kepler's 3rd law as updated here

The table shows the orbit data, in units ranging from AU, km, year. day, hour. With the 3rd law equation based on ratios not on the measured values, the orbits are confirmed to conform with the elliptical baseline for the groups of moons around planets.

Note an extra row is provided for the planets Mercury, Venus, Earth, and Mars. This row has the orbit data as km and days; these values do not conform to Kepler's 3rd law equating an orbit's radius and period. The description for using the equation must be clear. The equation is using ratios and is sensitive to the units of the original values.

5 Exoplanets

Using new technology, in recent decades astronomers were able to detect planets in orbit around distant stars; these are called exoplanets.

The orbit parameters were entered into an Excel spreadsheet for many of the systems with several exoplanets which had this data (radius and time per orbit).

reference for this spreadsheet of orbit data:

pdf-link: Exoplanet Data with check of Kepler's 3rd law as updated here

The table shows the orbit data, in units ranging from AU, year. day. With the 3rd law equation based on ratios not on the measured values, the individual orbits are confirmed to conform with the elliptical baseline for the groups of exoplanets around a distant star.

6 Conclusion from the spreadsheets

Kepler's 3rd law description uses a planet's semimajor axis and its period. In practice, those values of orbit radius and time cannot be used directly for all objects. Each orbit must be converted to a proportional value (with no units) for the equation.

After converting to a proportional factor the squared-cubed relationship can be confirmed for every orbit.
. This new description has been confirmed for all planets, moons, and asteroids in our solar system and for many exoplanet systems (not all exoplanets have data).

7 Conclusion

First, Celestial mechanics must recognize a collection of bodies having elliptical orbits around a primary will have proportional orbits.
The primary can be a star with orbiting planets and asteroids, or it can be a planet with orbiting moons.

Kepler's Third Law of Planetary Motion has an equation which apparently mixes units. This paper explained how this equation, with a new description, works with all planets (both around our Sun and around other stars), moons, and asteroids and their motion around their primary. Instead of entering a moon's or planet's orbit values directly, their proportional values for the system of bodies around a primary are entered into the equation.

Essentially, a body in rotation with other bodies also in an elliptical orbit around a common primary (either the Sun or a planet) will have its orbit conform with the others in the same radius squared and time cubed proportions as shown by the results of Kepler's 3rd law (with its amended description).
This revised description applies to planets, moons, and asteroids.
Kepler's Laws of Planetary Motion, are apparently limited to planets.

The scope must be extended beyond planets. The laws are for elliptical orbits. More than just planets have elliptical orbits.
The new name could be 3 Laws of Elliptical Motion.
This revised set of Laws applies to all bodies in an elliptical orbit around a primary. Only the 3rd law requires an amended description for its use
The equation in the 3rd law is the mathematical representation of the proportional relationship among the elliptical orbits in the system.
Kepler's 3 laws of motion apply to bodies whose motion is an ellipse.